Maróthi György Memorial Competition, October 31 – November 29, 2024.

– Rules –

Description

The Institute of Mathematics of the University of Debrecen announces a competition in Mathematics for BSc students of the University of Debrecen in their first or second year of studies during the fall semester of the academic year 2024/2025. The competition is individual, registration is not required. The list of problems is published at noon on October 31, on the web page of the Institute:

`https://math.unideb.hu`

Available at: Hallgatóknak » Tehetséggondozás » Versenyek » Maróthi György Memorial Competition

Organizers

dr. Boros Zoltán	*(Inst. Coordinator of Talent Management, Department of Analysis, UD)*
dr. Bessenyei Mihály	*(Competition Secretary, Department of Analysis, UM)*
dr. Nagy Ábris	*(Department of Geometry UD)*
dr. Tengely Szabolcs	*(Department of Algebra and Number Theory, UD)*

Sponsorship

Organizers thank the financial support by the Morgan Stanley Magyarország Elemző Kft.

Formal requirements

Solutions to distinct problems should be elaborated on separate sheets of paper. Write your name, major, year, neptun code and the number of the problem which is elaborated on that sheet to the top of the page. The pdf file of the hand written solutions have to be sent by email to Zoltán Boros and Mihály Bessenyei:

`zboros@science.unideb.hu` and `besse@science.unideb.hu`.

Deadline for submission: November 29 (Friday), 2024, 12:00.

Ethical regulation

Though all problems can be solved using standard college mathematics, you can use any additional sources if it is appropriately cited in your solution. Cooperation of the participants (with each other or with any other person on any platform) is not allowed. If such a cooperation is established, all involved participants will be disqualified.

Every participant will be notified of his/her result.

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1. Problem. *Find all real solutions of the equation*

$$x^3 = \left\lfloor 4x + \frac{3}{4} \right\rfloor,$$

where $\lfloor u \rfloor$ *denotes the the lower integer part (the „floor") of the real number* u*, i.e.,* $\lfloor u \rfloor = k$*, where* k *is the largest integer fulfilling* $k \leq u$*.*

(Posed by Boros Zoltán)

2. Problem. *Determine all real functions* $f\colon \mathbb{R} \to \mathbb{R}$ *that satisfy the functional equation*

$$f(x)f(x+y) = f(y)f(x-y)$$

for all $x, y \in \mathbb{R}$*.*

(Posed by Boros Zoltán)

3. Problem. *We draw squares on each side of the triangle* ABC *pointing outward and connect the center of every square to the vertex of the triangle which is not a vertex of that square. Prove that the three conencting lines intersect each other in a single point.*

(Posed by Nagy Ábris)

4. Problem. *One side of a triangle has length* $a = 6$*, the inner angle opposite to this side is* $\alpha = 60^\circ$*, and the radius of the inscribed circle is* $r = \sqrt{6} - \sqrt{3}$*. Compute the exact lengths of the missing sides and the inner angles of the triangle.*

(Posed by Nagy Ábris)

5. Problem. *Let* $L = \{-98, -82, -58, -34, 13, 16, 69, 75, 99\}$ *and* $L_k = \{a^k \mid a \in L\}$*,* $k = 1, 3$*. Show that neither* L_1*, nor* L_3 *can be arranged in magic squares of size* 3×3 *(where the sums of the rows, the sums of the columns and the sums of the two diagonals are the same).*

(Posed by Tengely Szabolcs)

6. Problem. *Show that there are infnitely many* 2×2 *matrices*

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

such that $a, b, c, d \in \mathbb{Z}$ *and*

$$A^2 - 2(A^T)^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Determine all matrices that satisfy the above equation and $abcd = 90$*.*

(Posed by Tengely Szabolcs)

Solution to each problem is evaluated up to 5 points.
The order of the problems need not indicate their difficulty.